The New Quantitative Trade Model: Equilibrium and Welfare Analysis

Dominick Bartelme
Konstantin Kucheryavyy

Yuyang Jiang
Andres Rodríguez-Clare

October 2023
Motivation

- **Quantitative trade models:**
  - Multiple sectors and intermediates
  - Roundabout production matching IO data
  - Caliendo-Parro or Baqae-Parro have CRS

- **Extend to allow for EES:**
  - New trade theory: EES due to love of variety
  - Empirical evidence for EES (recent): Costinot et. al. '19; BCDR; Lashkaripour and Lugovskyy '22; Bartelme et. al. '23; Breinlich et. al. '21
Examples in the Literature

- **BCDR '21**: industrial policy (PC)
- **Bartelme et. al. ’23**: trade shocks on growth (PC)
- **Lashkaripour and Lugovskyy ’22**: industrial policy (MC)
- **Breinlich et. al. ’21**: import shocks on exports (MC)

**Special cases:**
- **Krugman and Venables ’95**: core-periphery
- **Antras et. al. ’21**: trade policy
- **Caliendo et. al. ’21**: optimal trade policy (Melitz)
- **Baqaee and Farhi ’21**: local comparative statics (no trade)

**Background:**
- **KLR**: multi-sector gravity + EES, no intermediates
Model:
- Caliendo-Parro + EES in VA or GO, Small Open Economy
- $\varepsilon_k$ is the trade elasticity, $\theta_k$ is the scale elasticity

Uniqueness:
- Sufficient Uniqueness Condition (UC):
  $$\sum_s \theta_s \ell_{sk}^F \varepsilon_k < 1, \forall k$$
- Without IO: $\ell_{kk}^F = 1$ and $\ell_{sk}^F = 0$ for $s \neq k$ ⇒ KLR’s condition:
  $$\theta_k \varepsilon_k < 1, \forall k$$
- Proof is not yet complete for EES in GO and more than one sector with EES

Gains from Trade:
- With EES in VA, UC implies gains from trade
- With EES in GO, could have losses from trade even under UC
1. Motivation

2. Model
   2.1 Basic Assumptions
   2.2 Equilibrium

3. Characterization of Equilibrium

4. Gains from Trade
Basic Assumptions

- Home is SOE
- $K$ sectors indexed by $k = 1, \ldots, K$
- Armington assumption
- Perfect competition and sector-level EES
Basic Assumptions

\[ Q_k = \left( \alpha_k^{-\alpha_k} \prod_{s=1}^{K} \alpha_{sk}^{-\alpha_{sk}} \right) T_k L_k^{\alpha_k} \prod_{s=1}^{K} Q_{sk}^{\alpha_{sk}} \]

\[ \alpha_{sk} \in [0, 1], \quad \alpha_k + \sum_s \alpha_{sk} = 1, \quad \alpha_k > 0 \]

\[ \overline{T}_k = T_k L_k^{\alpha_k \gamma_k} \prod_{s=1}^{K} Q_{sk}^{\alpha_{sk} \nu_k} \]

\[ \nu_k < \frac{\alpha_k}{1 - \alpha_k} \]
Basic Assumptions

▶ If $\nu_k = 0$, then

$$Q_k = \left( \alpha_k^{-\alpha_k} \prod_{s=1}^{K} \alpha_{sk}^{-\alpha_{sk}} \right) T_k \cdot \left( L_k \cdot L_k^{\gamma_k} \right)^{\alpha_k} \prod_{s=1}^{K} Q_{sk}^{\alpha_{sk}}$$

▶ This is EES in VA, a natural framework for technological EES

▶ If $\gamma_k = \nu_k$, then

$$\overline{T}_k = \tilde{T}_k \cdot Q_k^{\frac{\gamma_k}{1+\gamma_k}}$$

▶ This is EES in gross output, and results from Krugman with

$$\gamma_k = \nu_k = \frac{1}{\sigma_k - 1}$$

where $\sigma_s$ is the EoS across domestic varieties
Basic Assumptions

- Composite consumption $\neq$ composite intermediate

\[
\lambda_k^C(p_k) = \frac{p_k^{-\varepsilon_k}}{p_k^{-\varepsilon_k} + [p_{k}^{C*}]^{-\varepsilon_k}}, \quad \lambda_k(p_k) = \frac{p_k^{-\varepsilon_k}}{p_k^{-\varepsilon_k} + [p_{k}^{I*}]^{-\varepsilon_k}}
\]

- Cobb-Douglas preferences across sectors

\[
C_k = \lambda_k^C(p_k)e_k \bar{w} \bar{L}
\]

- Isoelastic export revenues in sector $k$

\[
X_k = E_k p_k^{-\varepsilon_k}
\]
1. Motivation

2. Model
   2.1 Basic Assumptions
   2.2 Equilibrium

3. Characterization of Equilibrium

4. Gains from Trade
Equilibrium: Prices

Equilibrium prices given $\lambda^I_1, \ldots, \lambda^I_K$ and $L_1, \ldots, L_K$:

$$p_k = w \cdot \tilde{\xi}_k \cdot \prod_s \lambda_s^{\ell^F_{sk} - \delta_{sk}} \cdot \prod_s (T_s L_{-s} \theta_s)^{-\ell^F_{sk}},$$

where $\delta_{sk}$ indicator function for $s = k$,

$$\theta_s \equiv \alpha_s \gamma_s + (1 - \alpha_s) \nu_s$$

and

$$L^F \equiv (I - A(I + D\nu))^{-1} \quad \text{with} \quad A \equiv \{\alpha_{sk}\}, \ D \nu \equiv D\{\nu\}$$

capture forward linkages,

$$\ell^F_{sk} = -\partial \ln p_k / \partial \ln T_s$$
Equilibrium: Market Clearing

Market clearing condition in sector $k$ is

$$p_k Q_k = C_k + X_k + \lambda_k \sum_s P_k Q_{ks}$$

or, using $d_k \equiv (C_k + X_k)/w$,

$$L_k/\alpha_k = d_k + \lambda_k \sum_s \alpha_{ks} L_s/\alpha_s$$

Solving for $R_k \equiv L_k/\alpha_k$,

$$R_k = \sum_s \tilde{\ell}_{ks} B d_s$$

where

$$\tilde{\ell}^B \equiv (I - D_\lambda A)^{-1} \quad \text{with} \quad D_\lambda \equiv \mathcal{D}\{\lambda\}$$

captures backward linkages,

$$\tilde{\ell}_{ks}^B = \frac{\partial R_k}{\partial d_s}$$
An equilibrium is a wage $w$, prices $p$ and labor allocations $L$ that satisfy

$$p_k = \xi_k \cdot w \cdot \prod_s \left[ \lambda_s(p_s) \right]^{\ell_{sk}^F - \delta_{sk}} \cdot \prod_s L_s - \theta_s \ell_{sk}^F$$

$$\frac{L_k}{\alpha_k} = d_k(w, p_k) + \lambda_k(p_k) \sum_s \alpha_{ks} L_s / \alpha_s$$

$$\sum_k L_k = \bar{L}$$

We next *show* that there is a unique solution if

$$\sum_s \theta_s \ell_{sk}^F \varepsilon_k < 1, \forall k \quad UC$$
1. Motivation

2. Model
   2.1 Basic Assumptions
   2.2 Equilibrium

3. Characterization of Equilibrium

4. Gains from Trade
Characterization of Equilibrium

- **Step 1**: Take $w$ and $L$ as given and focus on $p$:
  
  \[ UC \implies \text{There exists a unique } p \]

  This leads to function $p(w, L)$

- **Step 2**: Take $w$ as given, focus on $L$:
  
  \[ UC \implies \text{There exists a unique } L \]

  This leads to labor demand $L(w)$

- **Step 3**: Focus on $w$:
  
  \[ UC \implies \text{There exists a unique } w \]
Characterization of Equilibrium

Steps 1 and 3 are straightforward, step 2 is challenging

- The goods market clearing condition gives a mapping $L \rightarrow L'$,

$$\frac{L'_k}{\alpha_k} = d_k (p_k (L)) + \lambda_k (p_k (L)) \sum_s \alpha_{ks} \frac{L'_s}{\alpha_s}$$

- Existence is proved by showing that (given UC) this mapping stays inside a rectangular region of $\mathbb{R}^{K_+}$

- To show uniqueness we use the “Index Theorem”
Index Theorem

- Index at a fixed point is $+1$ ($-1$) if $1 - F'(L) > 0$ ($< 0$)
- Generalization: index is $\text{sgn}(\det(I - J))$
- Index Theorem: sum of indices $= +1$
Key implication:

\[ \det (I - J) > 0 \text{ at any fixed point} \implies \text{fixed point is unique} \]

Basic idea: if a self-absorbing mapping is a local contraction mapping at each fixed point, then it has only one fixed point

Economics: supply curve cuts demand curve from below at every goods market equilibrium
Jacobian

- **UC** $\implies \det(I - J) > 0$ or $\rho(J) < 1$ for $J = \text{Jacobian of } L \to L'$ (in logs)
  mapping at a fixed point

- With no trade in intermediates,

$$
J_{|D\lambda=I} = D_R^{-1} \cdot \mathcal{L}^B \cdot \left\{ \frac{\partial d_k(p_k)}{\partial \ln p_r^{\varepsilon_r}} \right\} \cdot \left\{ \frac{\partial \ln p_k^{-\varepsilon_k}}{\partial \ln L_s} \right\}
$$

$$
\leq \mathcal{D} \left\{ \mathcal{L}^B D_{dt} \right\}^{-1} \cdot \mathcal{L}^B \cdot D_d \cdot D_\varepsilon \left[ \mathcal{L}^F \right]^T D_\theta \equiv \tilde{J}_{|D\lambda=I}
$$

- Thus $\rho\left(\tilde{J}_{|D\lambda=I}\right) < 1$ if max row sums of $D_\varepsilon \left[ \mathcal{L}^F \right]^T D_\theta$ are $< 1$, which is our **UC**,

$$
\sum_s \theta_s \ell^F_{sk} \varepsilon_k < 1, \forall k
$$
In the case with EES in VA we show that the UC,

\[ \sum_s \theta_s \ell_{sk}^F \varepsilon_k < 1, \forall k, \]

is sufficient for \( \rho(J) < 1 \) for any \( \lambda \)

**Intuition:** works with autarky in intermediates and 100% export demand, where strength of linkages and elasticity of demand are maximized

Still working on proof with EES in GO – so far we have it only for EES in one sector
Discussion

- With EES in VA \((\nu_k = 0, \forall k)\) and \(\gamma_k = \gamma, \forall k\):
  
  \[\nu_k = 0, \forall k \implies \theta_k = \gamma\alpha_k \text{ and } \ell^F_{sk} = \ell^B_{sk}, \forall s, k \text{ so UC becomes}\]
  
  \[\gamma \sum_s \alpha_s \ell^B_{sk} \varepsilon_k = \gamma \varepsilon_k < 1, \forall k,\]

  where we have used \(\sum_s \alpha_s \ell^B_{sk} = 1\)

- This is same UC in KLR for case without IO if \(\gamma_k = \gamma, \forall k\)

- With EES in GO \((\nu_k = \gamma_k, \forall k)\) and \(\gamma_k = \gamma, \forall k\) then UC becomes

  \[\varepsilon_k \leq \frac{1}{\gamma \sum_s \ell^F_{sk}}\]

- EES in GO leads to increased amplification relative to EES in VA
\[ \varepsilon_k \leq \frac{1}{\gamma \max_k \sum_s \ell_{sk}^F} \text{ for US (Motor Vehicles)} \]
### Table 1: Maximum TE that Satisfies the Uniqueness Condition, Selected Sectors

<table>
<thead>
<tr>
<th>Sector</th>
<th>Max $\epsilon_k$ (1)</th>
<th>Max $\epsilon_k$, 10\textsuperscript{th} pctile (2)</th>
<th>Max $\epsilon_k$, US (3)</th>
<th>Max $\epsilon_k$, Avg. IO (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>2.8</td>
<td>3.3</td>
<td>4.2</td>
<td>5.3</td>
</tr>
<tr>
<td>Mining</td>
<td>3</td>
<td>3.5</td>
<td>5</td>
<td>6.1</td>
</tr>
<tr>
<td>Textiles</td>
<td>2.2</td>
<td>2.7</td>
<td>4.4</td>
<td>3.5</td>
</tr>
<tr>
<td>Chemical Products</td>
<td>2.1</td>
<td>2.6</td>
<td>3.9</td>
<td>3.5</td>
</tr>
<tr>
<td>Basic Metals</td>
<td>1.8</td>
<td>2.3</td>
<td>3.2</td>
<td>3.1</td>
</tr>
<tr>
<td>Machinery &amp; Equipment</td>
<td>1.9</td>
<td>2.6</td>
<td>3.8</td>
<td>3.4</td>
</tr>
<tr>
<td>Motor Vehicles</td>
<td>1.9</td>
<td>2.2</td>
<td>2.9</td>
<td>2.9</td>
</tr>
<tr>
<td>Construction</td>
<td>2.3</td>
<td>2.7</td>
<td>4.5</td>
<td>4</td>
</tr>
<tr>
<td>Wholesale/Retail Trade</td>
<td>3.6</td>
<td>3.9</td>
<td>5.8</td>
<td>5.5</td>
</tr>
<tr>
<td>Finance &amp; Insurance</td>
<td>2.4</td>
<td>4.4</td>
<td>4.9</td>
<td>5.3</td>
</tr>
<tr>
<td>Education</td>
<td>3.9</td>
<td>5.1</td>
<td>6.5</td>
<td>6.6</td>
</tr>
<tr>
<td>Avg. Ratio w/ column 1</td>
<td>1</td>
<td>0.8</td>
<td>0.5</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Assumes $\gamma_k = \upsilon_k = 0.1$, $\forall k$. 
1. Motivation

2. Model
   2.1 Basic Assumptions
   2.2 Equilibrium

3. Characterization of Equilibrium

4. Gains from Trade
Gains from Trade

To simplify, use $\lambda^C_k = \lambda^I_k = \lambda_k$. We then have

$$GT = \prod_k (\lambda_k)^{-\frac{\psi^F_k}{\varepsilon_k}} \times \prod_k \left( \frac{L_k}{\alpha_k \psi^B_k \bar{L}} \right)^{\theta_k \psi^F_k}$$

where $\psi^F_k \equiv \sum_s \ell^F_{ks} e_s$ and $\psi^B_k \equiv \sum_s \ell^B_{ks} e_s$ are forward and (closed economy) backward Domar weights (Baqee and Farhi '21)

Combined with

$$L_k = \alpha_k \sum_s \tilde{\ell}^B_{ks} (\lambda_s e_s + x_s) \bar{L} \geq \alpha_k \sum_s \tilde{\ell}^B_{ks} \lambda_s e_s \bar{L}$$

with $x_s \equiv X_s/w \bar{L}$, we then have

$$GT \geq GT^\ast (\lambda) \equiv \prod_k \lambda_k^{-\frac{\psi^F_k}{\varepsilon_k}} \times \prod_k \left( \frac{\sum_r \tilde{\ell}^B_{kr} (\lambda) e_r \lambda_r}{\psi^B_k} \right)^{\theta_k \psi^F_k}$$
KLR showed that

\[ \gamma_k \epsilon_k < 1 \implies P_k \downarrow \text{ as } \lambda_k \downarrow \text{ below one} \implies GT^* > 1 \]

Condition \( \gamma_k \epsilon_k < 1 \) also guarantees uniqueness

Does UC also guarantee \( GT^* > 1 \) in the current setting?
We show that $G^T_\ast (\lambda)$ is strictly (log-log) convex in $\lambda$ so if

$$-\frac{\partial \ln G^T_\ast}{\partial \ln \lambda_i} \bigg|_{\text{Autky}} \geq 0$$

for all $i$ then $G^T_\ast > 1$ for any trade pattern.
Gains at Autarky

We have

\[ -\frac{\partial \ln GT^*}{\partial \ln \lambda_i} \bigg|_{\text{Autky}} = \frac{\psi_i^F}{\varepsilon_i} \frac{\partial \ln L_k^*}{\partial \ln \lambda_i} \bigg|_{\text{ACR}} - \sum_k \psi_k^F \theta_k \frac{\partial \ln L_k^*}{\partial \ln \lambda_i} \bigg|_{\text{EES}} \]

and

\[ \frac{\partial \ln L_k^*}{\partial \ln \lambda_i} \bigg|_{\text{Autky}} = \frac{\ell_{ki}^B \psi_i^B}{\psi_k^B}. \]

Using \( \Psi_k \equiv \psi_k^F / \psi_k^B \) for “distortion centrality of sector \( k \)” (Liu ’19) and \( m_i \equiv (1 - \lambda_i) \left[ e_i + \sum_s \alpha_{is} R_s / \bar{L} \right] \) for imports in sector \( i \) as a share of GDP, we have

\[ \frac{\partial \ln GT^*}{\partial m_i} \bigg|_{\text{Autky}} = \frac{\psi_i}{\varepsilon_i} \frac{\partial \ln L_k^*}{\partial \ln \lambda_i} \bigg|_{\text{ACR}} - \sum_k \theta_k \Psi_k \ell_{ki}^B \bigg|_{\text{EES}}. \]
Gains at Autarky: EES in VA

\[ \frac{\partial \ln \text{GT}^*}{\partial m_i} \bigg|_{\text{Autky}} = \frac{\Psi_i}{\epsilon_i} - \sum_k \theta_k \psi_k \ell_{ki}^B \]

If EES in VA then \( \mathcal{L}^B = \mathcal{L}^F \) and so \( \psi_k^F = \psi_k^B, \forall k \) plus the UC \( \implies \)

\[ \frac{\partial \ln \text{GT}^*}{\partial m_i} \bigg|_{\text{Autky}} = \frac{1}{\epsilon_i} - \sum_k \theta_k \ell_{ki} > 0, \forall i \implies \text{GT}^* > 1 \]
Gains at Autarky: EES in GO

\[
\left. \frac{\partial \ln G T^*}{\partial m_i} \right|_{\text{Autky}} = \frac{\Psi_i}{\varepsilon_i} - \sum_k \theta_k \Psi_k \ell_{ki}^B
\]

- If EES in GO then \( \mathcal{L}^B \neq \mathcal{L}^F \) so UC can hold while \( -\left. \frac{\partial \ln G T^*}{\partial m_i} \right|_{\text{Autky}} < 0 \)

- Assuming \( \varepsilon_i = \varepsilon, \forall i \) and \( \theta_k = \theta, \forall k \) then

\[
\left. \frac{\partial \ln G T^*}{\partial m_i} \right|_{\text{Autky}} = \frac{1}{\varepsilon} \Psi_i - \theta \sum_k \Psi_k \ell_{ki}^B
\]

\( \Rightarrow \) imports in sectors with low distortion centrality but high backward distortion centrality can cause welfare losses
Adding Exports

† To a first order (around autarky), the gains from trade are

\[
\ln GT \approx \sum_k \frac{\psi_k}{\varepsilon_k} m_k + \sum_{k,s} \theta_k \psi_k \ell_{ks}^B (x_s - m_s)
\]

† If EES in VA then

\[
\ln GT \approx \sum_k \frac{m_k}{\varepsilon_k} + \sum_s \tilde{\gamma}_s (x_s - m_s),
\]

where \(\tilde{\gamma}_s \equiv \sum_k \gamma_k \alpha_k \ell_{ks}^B\)

† Higher gains if specialize in sectors with high backward EES
Adding Exports

To a first order (around autarky), the gains from trade are

\[ \ln GT \approx \sum_k \frac{\psi_k}{\varepsilon_k} m_k + \sum_{k,s} \theta_k \psi_k \ell_{ks}^B (x_s - m_s), \]

With common elasticities then

\[ \ln GT \approx \frac{1}{\varepsilon} \sum_k m_k + \frac{1}{\varepsilon} \sum_k (\psi_k - 1) m_k + \theta \sum_{k,s} \psi_k \ell_{ks}^B (x_s - m_s) \]

- DC gains higher if imports mostly in sectors with high DC, which tend to be upstream (Liu '19)
- EES gains higher if specialize in sectors with high backward DC
Quantitative Implications

- What are the implications of **uniform** EES on GT?
- Use world average IO matrix and compute \( \ln \) GT, ACR, DC and EES gains.
- Regressing ACR gains on \( \ln \) GT gives share of variance of \( \ln \) GT explained by ACR gains
Gains ACR vs aggregate

Slope=0.68
Conclusions

- Incorporate EES into quantitative trade models
- Open computational black box: equilibrium and welfare properties
- Sufficient condition for uniqueness
  \[ \sum_s \theta_s \ell_{sk}^F \epsilon_k < 1, \forall k \]
- Neets simpler condition \( \theta_k \epsilon_k < 1 \) without IO
- IO makes (sufficient) upper bound on \( \theta' \)s much tighter
- UC ensures gains if EES in VA, but not if EES in GO
- Larger GT with specialization in sectors with higher EES upstream
With sector wedges $\mu_k$ we then have

\[
d \ln G_T = \sum_k \psi_k^F d \ln \lambda_k^{-1/\varepsilon_k} + \sum_k \eta_k d \ln L_k
\]

where

\[
\eta_k \equiv \frac{\left(1 + \frac{\mu_k - 1}{\alpha_k}\right) L_k / \bar{L}}{\sum_s \left(1 + \frac{\mu_s - 1}{\alpha_s}\right) L_s / \bar{L}}
\]

- Same $\psi_k^F$ as above with $\mu_k - 1$ instead of $\nu_k$
- But different from the case with EES,

\[
d \ln G_T = \sum_k \psi_k^F d \ln \lambda_k^{-1/\varepsilon_k} + \sum_k \theta_k \psi_k^F d \ln L_k
\]
Wedges vs EES

With no IO, $\psi^F_k = e_k$ and $\alpha_k = 1$, so

$$EES: \quad d \ln \text{GT}|_{Autky} = \sum_k e_k d \ln \lambda_k^{-1/\varepsilon_k} + \sum_k \theta_k \frac{dL_k}{\bar{L}}$$

Markups: $d \ln \text{GT} = \sum_k e_k d \ln \lambda_k^{-1/\varepsilon_k} + \sum_k \frac{\mu_k}{\bar{\mu}} \frac{dL_k}{\bar{L}}$, $\bar{\mu} \equiv \sum_s \mu_s L_s/\bar{L}$

With uniform EES or wedges then result is the same, ACR

Equivalence is broken with IO:

$$EES: \quad d \ln \text{GT} = \sum_k \psi^F_k d \ln \lambda_k^{-1/\varepsilon_k} + \sum_k \theta_k \psi^F_k d \ln L_k$$

Markups: $d \ln \text{GT} = \sum_k \psi^F_k d \ln \lambda_k^{-1/\varepsilon_k} + \sum_k \eta_k d \ln L_k$